

APM462H1Y: Nonlinear Optimization
Summer 2025
University of Toronto

I. Instructor and Teaching Assistants

Course Instructor

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I. Course Overview

Course Description

An introduction to first and second order conditions for finite and infinite dimensional optimization problems with mention of available software. Topics include Lagrange multipliers, Kuhn-Tucker conditions, convexity and calculus of variations. Basic numerical search methods and software packages which implement them will be discussed.

Prerequisites

([MAT223H1](#), [MAT224H1](#)) / [MAT247H1](#), [MAT235Y1](#) / [MAT237Y1](#) / [MAT257Y1](#)

Course Objectives

Being able to solve finite and infinite dimensional optimization problems with constraints.

Textbooks/ Course Readings

For the first part of the course, we will use the textbook *Linear and Nonlinear Programming* by Luenberger and Ye (5th edition). A digital copy of the book is available through the U of T library system.

For the second part of the course we will loosely follow the textbook *The Calculus of Variations* by van Brunt. A digital copy of the book is available through the U of T library system.

How this course is organized

The course, including the final exam, is fully online, i.e. physical presence at the St. George campus is not necessary. All lectures, tutorials, office hours etc will be held on Zoom.

Lectures: LEC5101 Tuesday 6.00 to 8.00 pm EDT and Thursday 6.00 to 7.00 pm EDT, LEC5102 Tuesday 5.00 to 6.00 pm EDT and Thursday 4.00 to 6.00 pm EDT.

Tutorials: TUT0101 and TUT0102 Wednesday 9.00 to 10.00 am EDT, TUT0201 and TUT0202 Wednesday 10.00 to 11.00 am EDT, TUT5101 and TUT5102 Thursday 5.00 to 6.00 pm EDT. Tutorials start in week 2 of the semester, i.e. on May 13.

Office hours: Instructor office hours take place on Monday between 10.00 and 11.00 am (starting May 13). TA office hours will also happen weekly; the time will be announced later.

All lectures will be recorded and will be available for students to rewatch until after the final project is due. Recording of the tutorials is up to discretion of the TA. Office hours will not be recorded.

Information about the course, assignments, announcements etc will be delivered through Quercus. Assignments will be submitted on Crowdmark.

The final project will be submitted on Quercus – a plagiarism tool will be used to check submissions (see below).

Technical Requirements

In order to participate in this course, students will be required to have:

- Reliable internet access. It is recommended that students have a high speed broadband connection (LAN, Cable, or DSL) with a minimum download speed of 5 Mbps.
- A computer satisfying the minimum technical requirements (<https://www.vicereprodstudents.utoronto.ca/student-policies-guidelines/tech-requirements-online-learning/>)

Other recommended items include headphones, microphone, webcam, and a tablet or printer.

If you are facing financial hardship, you are encouraged to contact your college or divisional registrar (<https://future.utoronto.ca/current-students/registrars/>) to apply for an emergency bursary.

II. Evaluation/ Grading Scheme

Mark Breakdown

The course has two marked components, the six (6) assignments (A) and the final project (F). Each assignment contributes equally to (A), however, the assignment with the lowest grade will not count towards (A).

The final course grade will be computed as follows:

If F is greater or equal to 50% then the final course grade is $0.4*A + 0.6*F$;

If F is less than 50% the final course grade is $1.0*F$.

Assignments

There will be six (6) assignments, the one with the lowest grade will be dropped. Tentative due dates can be found in the lecture schedule at the end of this document; assignments will be due usually at the end of the week on Saturday at 11.59 pm. Assignments will be submitted through Crowdmark. The submissions may be in handwriting or may be written in LaTeX. Only selected exercises from the Assignments will be graded. Each exercise will be graded out of 10 points.

Final Project

The final project will be held during the final assessment period in August 2025. Information about the format will be provided during the summer semester. The final project will be like a homework assignment, just longer and more complex. The final project will be open book. It will be submitted on Quercus and must be written with LaTeX. Submissions not written with LaTeX will receive a grade of 0%, unless the Course Instructor explicitly gave the student permission to use another type setting language. Submissions in handwriting, Microsoft Word or Google Docs are not acceptable.

There will be additional office hours before the final project is due where TAs will provide feedback on submissions and help with any issues with LaTeX.

III. Course Policies

Policy on Missed Term Work

Students may submit assignments and the final project up to 8 hours late after the deadline that they were due without penalty. Assignments submitted later will automatically be graded with 0%, unless there are legitimate, documented reasons beyond a student's control, such as medical issues (see below).

Students who are absent from class for prolonged periods and who require consideration for missed academic work should contact the instructor and verify their absence(s) through either the Absence Declaration tool, Verification of Illness or Injury (VOI) form, College Registrar Letter, or Letter of Academic Accommodation from Accessibility Services, as appropriate to their situation.

<https://www.artsci.utoronto.ca/current/academics/student-absences>

The absence declaration can be used once per term. Outside of the one time absence declaration use, students must adhere to the alternate processes for absences listed above, as well as the missed work policy as set out in each course's syllabus.

Re-marking Policy

A student who believes an individual item of work has been incorrectly or unfairly marked may ask the person who marked it for a re-evaluation. With evidence to back their appeal, students should make such requests as soon as reasonably possible after receiving the work back, but no later than 2 weeks after it was returned.

Plagiarism Detection Tool

A plagiarism detection tool may be used for detecting plagiarism in some of the written work submitted in this course. Normally, students will be required to submit their written work to the university's plagiarism detection tool for a review of textual similarity and detection of possible plagiarism. In doing so, students will allow their work to be included as source documents in the tool's reference database, where they will be used solely for the purpose of detecting plagiarism. The terms that apply to the University's use of the this tool's service are described on the Centre for Teaching Support and Innovation web site:

<https://teaching.utoronto.ca/resources/plagiarism-detection/>

(Generative) AI

The use of generative artificial intelligence (AI) tools is strictly prohibited in all course assessments unless explicitly stated otherwise by the instructor. This includes, but is not limited to, ChatGPT, GitHub Microsoft Copilot, AI Tutor and Teacher's Assistant Pro, and open-source models that you have trained and/or deployed yourself. You may not interact with, nor copy, paraphrase, or adapt any content from any generative AI for the purpose of completing assignments in this course. Use of generative AI will be considered use of an unauthorized aid, which is a form of academic misconduct under the [Code of Behaviour on Academic Matters](#).

This course policy is designed to promote your learning and intellectual development and to ensure that our evaluations are a fair and accurate assessment of your learning. Though it may be tempting to use generative AI to assist you when completing your assignments, this will simply inhibit your learning. If the work you submit is essentially the output of generative AI, then what have you learned and what value are you adding? Think of it this way: if a potential employer or supervisor can get as much from an AI tool as what you're able to do yourself, then why should they hire you at all? You should aim to understand course content at a level that far exceeds what an automated tool can achieve. Our course—and in particular, each assignment—is designed to help you attain true mastery of the course content. If you have questions or are stuck, please come to office hours, where we'll be happy to help!

Email Policy

Should you have a question that is not answered on the course site on Quercus (please check there first!) please note that all communications with the Course Instructor or TA's must be sent from your official utoronto email address, with the course number included in the subject line. If these instructions are not followed, your email may not be responded to. Please write in a professional manner.

IV. Institutional Policies and Support

Academic Integrity

All suspected cases of academic dishonesty will be investigated following procedures outlined in the Code of Behaviour on Academic Matters (<https://governingcouncil.utoronto.ca/secretariat/policies/codebehaviour-academic-matters-july-1-2019>). If you have questions or concerns about what constitutes appropriate academic behaviour or appropriate research and citation methods, please reach out to your Course Instructor. Note that you are expected to seek out additional information on academic integrity from me or from other institutional resources (for example, the University of Toronto website on Academic Integrity <http://academicintegrity.utoronto.ca/>).

Recording of Online Course Materials

Notice of Video Recording and Sharing (Download and Re-Use Prohibited)

This course, including your participation, will be recorded on video and will be available to students in the course for viewing remotely and after each session.

Course videos and materials belong to your instructor, the University, and/or other sources depending on the specific facts of each situation and are protected by copyright. Do not download, copy, or share any course or student materials or videos without the explicit permission of the instructor.

For questions about recording and use of videos in which you appear please contact your instructor.

Accessibility

The University provides academic accommodations for students with disabilities in accordance with the terms of the Ontario Human Rights Code. This occurs through a collaborative process that acknowledges a collective obligation to develop an accessible learning environment that both meets the needs of students and preserves the essential academic requirements of the University's courses and programs.

Students with diverse learning styles and needs are welcome in this course. If you have a disability that may require accommodations, please feel free to approach your Course Instructor and/or the Accessibility Services office as soon as possible. The sooner you let us know your needs the quicker we can assist you in achieving your learning goals in this course.

Link to Accessibility Services website: <https://studentlife.utoronto.ca/departments/accessibility-services/>

Equity, Diversity and Inclusion

The University of Toronto is committed to equity, human rights and respect for diversity. All members of the learning environment in this course should strive to create an atmosphere of mutual respect where all members of our community can express themselves, engage with each other, and respect one another's differences. U of T does not condone discrimination or harassment against any persons or communities.

Important Academic Dates & Deadlines

The academic dates include enrolment dates, drop deadlines, exam periods, petition deadlines and more.

<https://www.artsci.utoronto.ca/current/dates-deadlines/academic-dates>

Other Academic and Personal Supports

- U of T Libraries <https://onesearch.library.utoronto.ca/>
- Student Code of Conduct: <https://governingcouncil.utoronto.ca/secretariat/policies/code-studentconduct-december-13-2019>
- Feeling Distressed? <https://studentlife.utoronto.ca/task/support-when-you-feel-distressed/>
- Academic Success Centre <https://studentlife.utoronto.ca/departments/academic-success/>
- College/Faculty Registrars <https://future.utoronto.ca/current-students/registrars/>

V. Schedule of Course

LY refers to Luenberger and Ye; vB refers to van Brunt.

| Week | Start date | End date | Lectures | Tutorials | Assignments |
|------|--------------|--------------|--|---|-----------------------|
| 1 | May 5, 2025 | May 11, 2025 | Introduction, syllabus, course overview Motivation Unconstrained optimization: Necessary and sufficient conditions (LY 7.1-7.3) Taylors theorem/ Mean-value theorem (LY A.6) | No tutorials | |
| 2 | May 12, 2025 | May 18, 2025 | Convex functions (LY 7.4) Convex optimization (LY 7.5) | Tutorial 1: Discuss Assignment 1 | |
| 3 | May 19, 2025 | May 25, 2025 | Descent algorithms: Basics, convergence theorem (LY 7.6) Method of Steepest Descent, quadratic case (LY 8.2) | Tutorial 2: Discuss Assignment 1 | Assignment 1 due |
| 4 | May 26, 2025 | June 1, 2025 | Conjugate direction method (LY 9.1-9.3) | Tutorial 3: Discuss solutions to Assignment 1 | Assignment 1 returned |

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| | | | Constrained optimization: Tangent plane (LY 11.1) | | |
| 5 | June 2, 2025 | June 8, 2025 | Constrained optimization: Necessary and sufficient conditions for equality constraints (LY 11.2-11.4) | Tutorial 4: Discuss Assignment 2 | Assignment 2 due |
| 6 | June 9, 2025 | June 15, 2025 | Constrained optimization: Necessary and sufficient conditions for inequality constraints (LY 11.5) | Tutorial 5: Discuss solutions to Assignment 2/ Assignment 3 | Assignment 2 returned |
| 7 | June 16, 2025 | June 22, 2025 | No lectures/ tutorials | | Assignment 3 due |
| 8 | June 23, 2025 | June 29, 2025 | | | Assignment 3 returned |
| 9 | June 30, 2025 | July 6, 2025 | No lecture on July 1 Lagrangian duality (LY 11.7-11.8) | Tutorial 6: Discuss solutions to Assignment 3 | |
| 10 | July 7, 2025 | July 13, 2025 | Lagrangian duality (LY 11.7-11.8) | Tutorial 7: Discuss Assignment 4 | Assignment 4 due |
| 11 | July 14, 2025 | July 20, 2025 | Calculus of Variations: Intro (vB 1) First variation (vB 2.1,2.2) Special cases, Generalizations (vB 2.3) | Tutorial 8: Discuss solutions to Assignment 4 | Assignment 4 returned |
| 12 | July 21, 2025 | July 27, 2025 | Second variation (vB 10.1,10.2) Convex functionals (vB 10.7) | Tutorial 9: Discuss Assignment 5 | Assignment 5 due |
| 13 | July 28, 2025 | August 3, 2025 | Isoperimetric problems (vB 4) | Tutorial 10: Discuss solutions to Assignment 5 | Assignment 5 returned |
| 14 | August 4, 2025 | August 10, 2025 | Isoperimetric problems (vB 6.1) Holonomic problems (vB 6.1) | Tutorial 11: Discuss Assignment 6 | Assignment 6 due |
| 15 | August 11, 2025 | August 17, 2025 | Holonomic problems (vB 6.1) No lecture on August 14 | No tutorials | Assignment 6 returned |
| 16 | August 18, 2025 | August 24, 2025 | No lectures/ tutorials | | Final Project due |
| 17 | August 25, 2025 | August 31, 2025 | | | Final Project returned |